

Decision-making in the production chain of a firm based on a dynamic planning model with backward recursion

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ABSTRACT

The inspection of spare parts and finished products in current production processes often struggles to meet the demands of various production scenarios. To minimize production costs across different situations, this paper examines six production scenarios. Using a dynamic programming model, we optimize the cost-minimization goal through an inverse recursive solution for the four phases of enterprise production. The paper analyzes whether to inspect parts and finished products and whether to disassemble nonconforming products at each stage of production under different conditions.

KEYWORDS

dynamic production decision-making; uncertainty management; dynamic planning models

1 Introduction

Enterprises often face complex decisions in production processes to minimize costs and improve profitability. However, current theoretical and empirical research on production decision-making has limitations, particularly in dynamic environments with varying production scenarios. Davide et al. (2021) and Anna et al. (2021) highlight that many decision-making models rely on complex mathematical calculations, making them difficult for managers to understand and apply in practice, thus limiting their usefulness in dynamic production settings. Additionally, Adriane et al. (2022) and Ahmed et al. (2020) emphasize the importance of coordinating multiple stakeholders in production decisions, especially for quality inspection and production strategies, an area often underexplored in existing studies.

Further, Demirel et al. (2019) and Yu et al. (2019) argue that production environments are subject to dynamic factors like demand fluctuations and supply chain instability, but most models assume static conditions, making it hard for companies to adjust quickly. Dynamic planning models, as suggested by Teece (2018), Bocken and Geradts (2019), are better suited to handle uncertainties like demand fluctuation and supply chain disruption, and can optimize resource allocation, inventory, and facility siting. However, data quality issues, as noted by Levenson and Fink (2017), limit the application of decision-making models based on historical data. Dynamic programming models are particularly useful in such cases, though further exploration is needed for specific production contexts like spare parts and finished product inspection.

Our study addresses these gaps by developing a dynamic programming model that accounts for factors such as demand fluctuations, supply chain changes, and multi-stakeholder coordination. We propose a flexible decision-making framework to help companies adapt their inspection strategies to changing market conditions, enhancing competitiveness and flexibility.

The structure of our research is as follows: first, we introduce the dynamic planning model in the theory section, followed by the experimental section detailing the model's establishment process, the presentation of calculation results, and a concluding summary with a future outlook.

2 Relevant theories

Dynamic Programming (DP) is an optimization method for solving complex problems by breaking them into smaller, overlapping subproblems. The core idea is to decompose a large problem into subproblems and construct the optimal solution recursively. This reduces computational complexity and avoids repetition, providing an efficient solution for large datasets and variable conditions.

Let's set up a large problem S can be decomposed into several subproblems s that dynamic programming transforms the problem into solving the optimal solution to a set of recursive equations. If $V(S)$ represents the optimal solution under the subproblems the optimal solution under the subproblem, then the goal of dynamic programming is to obtain the optimal solution of the overall problem by combining the optimal solutions of all the subproblems $V(S)$. Suppose that $V(S)$ Denote the:

$$V(s) = \max\{C(s) + \sum_{i=1}^n V(s_i)\} \quad (1)$$

Among them, $C(s)$ is the benefit or cost of the decision in the state, the benefit or cost of the state, and s_i denotes the benefit or cost of the decision in the associated subproblems. By such recursive computation, dynamic programming is able to find the globally optimal solution while avoiding repeated computations.

In the optimization of enterprise resource allocation, the total amount of resources can be set to be R , and the marginal benefit from each resource unit is b_i . Assume that each decision option D_i requires a resource of r_i and its benefit is p_i , then the firm's objective is to choose the combination of options that maximizes total return within the total resource R constraints, to select the combination of options that maximizes the total return. This is expressed in terms of the dynamic programming formula:

$$\max \sum_{i=1}^n (p_i + b_i \cdot r_i) \times D_i \quad (2)$$

The constraints are:

$$\sum_{i=1}^n r_i \times D_i \leq R \quad (3)$$

This formula allows solving each sub-problem step by step within the dynamic programming framework, aligning resource allocation with revenue expectations to construct an optimal resource allocation plan. Future demand fluctuations are modeled probabilistically, assuming a normal distribution $N(\mu, \sigma^2)$, enabling adjustments to the strategy under uncertainty and improving the enterprise's adaptability and competitiveness in a dynamic market environment.

3 Experimentation

Quality inspection and resource optimization are key to production efficiency. The complexity of production makes it difficult to decide on testing spare parts and finished products or handling non-conforming products to minimize costs. Companies must consider inspection costs and manage uncertainties like defect rate fluctuations, market demand, and supply chain instability. To address this, we simulated the production process, accounting for potential issues in spare parts and finished product inspections.

Two types of spare parts and finished product defect rates are known.

First, spare parts are either tested or directly assembled if untested. Unqualified parts are discarded. Then, assembled products are tested; if untested, they go to market, but only qualified ones are sold if tested. Unqualified finished products may be disassembled; if not, they are discarded. For unqualified products returned by users, the company exchanges them, incurring exchange losses. This process is repeated for returned non-conforming products.

This involves a number of decision-making aspects of the production process of the enterprise. Among other things, the goal is to maximize the combined profit generated by the enterprise during the production, testing and dismantling processes, while meeting the quality requirements of production.

Create an objective function where net profit can be obtained by subtracting total costs from total revenue:

$$P = R - C \quad (4)$$

Where P stands for Net Profit, R stands for Total Revenue and C stands for Total Cost.

Assuming total revenue is easily accessible, the goal is to minimize production costs. Total costs include parts purchase, assembly, inspection, exchange losses, and dismantling costs. Total revenue depends on the number of finished products sold and the defect rate. Decision variables include whether to test spare parts and finished products, and whether to dismantle substandard finished products.

3.1 Disassembly Decision Modeling Establishment

In production, defective products are not always discarded immediately. Instead, parts may be selectively disassembled for reuse. Disassembly allows spare parts to be reintroduced into production, but it incurs costs, including both production and dismantling expenses. This paper analyzes the decision-making model for disassembling defective products.

When the cost of dismantling is lower than the revenue generated by recycling spare parts, dismantling is carried out, otherwise it is discarded.

In this question, the formula for dismantling costs is

$$C_{dis} = dis_cost \times p_{prod} \times num_roducts \quad (5)$$

Among them.

dis_cost : Cost of dismantling each non-conforming finished product.

p_{prod} : The defective rate of the finished product, i.e. the proportion of the finished product that is substandard.

$num_roducts$: The number of finished products produced.

The formula for calculating the recovery benefit is

$$V_{rec} = sale_price \times p_{prod} \times num_roducts \quad (6)$$

Among them.

sale_price: The selling price of the finished product.

If $V_{rec} > C_{dis}$, then disassembly is recommended; otherwise, discard.

3.2 Construction of recursive equations

After defining the decision criteria for disassembling or discarding non-conforming products, we integrate this strategy into a dynamic planning framework using a recursive model. This model connects production stages and optimizes decision-making through cost-benefit analysis. The disassembly decision plays a key role, with backward and forward state transfers helping calculate optimal solutions across stages, covering spare parts inspection, finished product inspection, and processing. This provides a clear model and optimization direction for the experiment, as shown in Figure 1.

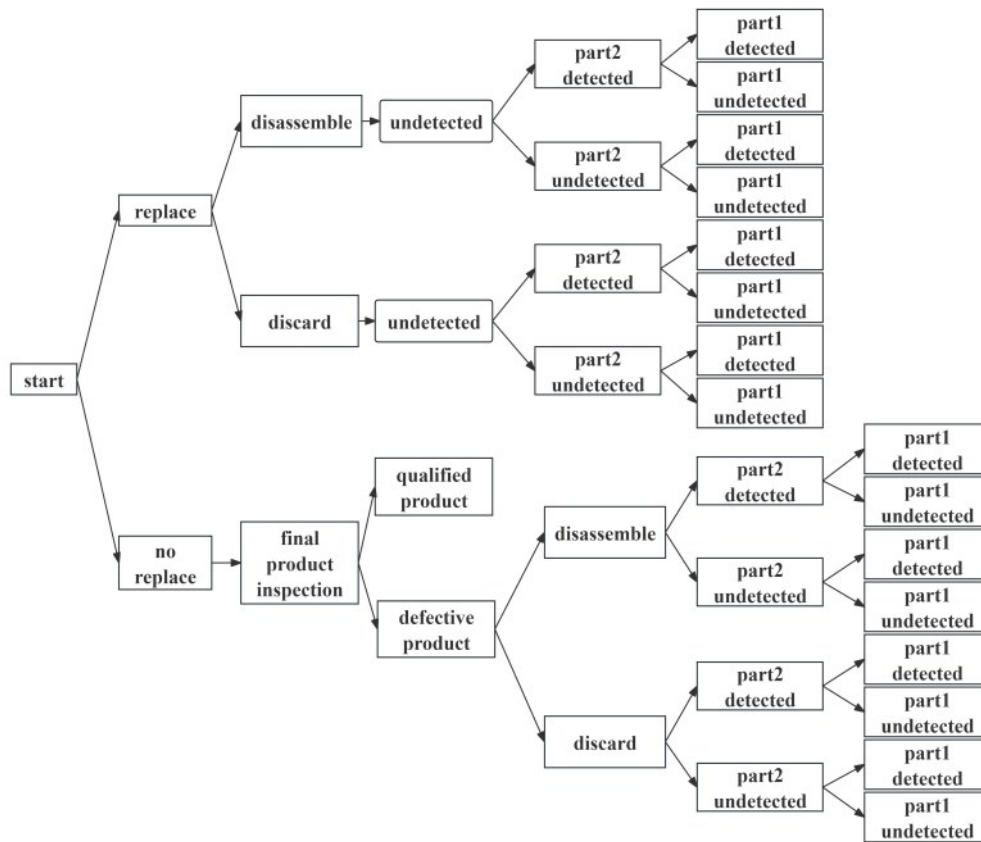


Figure 1 Backtracking solution process for dynamic programming

This recursive method highlights dependencies between stages, enabling enterprises to simulate strategies, quickly find optimal plans, reduce computational complexity, reuse results, and adapt to dynamic changes like defect rate fluctuations or inspection costs.

The dynamic programming stages break the complex problem into subproblems, each representing a decision stage. Stages 1-4 involve deciding whether to test part 1, part 2, the finished product, and how to dispose of nonconforming finished products (disassemble or discard).

And then define the state, in this question, we can define the following variables: S_{1-4} , respectively, are whether to detect part 1, whether to detect part 2, whether to detect the finished product, and throw the unqualified finished product or disassemble the unqualified finished product, and the state values are $\{0,1\}$. Among S_4 In this case, discard is 1 and disassemble is 0.

State transfer equations are derived backward to limit computation size and avoid redundancy. Forward derivation would involve more paths, leading to unnecessary calculations. Each stage has clear transfer equations, with results depending on subsequent states. Backpropagation starts from the final stage, resolving dependencies step by step.

Stage 4: Disposal of non-conforming finished products, including both disassembly and disposal options.

If you choose to dismantle the non-conforming finished product, the cost is:

$$Cost(S_4 = 1) = p_{prod} \times num_{product} \times disassemble_{cost} \quad (7)$$

Of which $disassemble_{cost}$ is the cost of dismantling nonconforming finished products.

If you choose to discard the nonconforming finished product, the cost is:

$$Cost(S_4 = 0) = 0 \quad (8)$$

(No cost is incurred for discarding nonconforming finished goods).

State transfer equations:

$$Cost(S_4) = \min(Cost(S_4 = 1), Cost(S_4 = 0)) \quad (9)$$

Stage 3: Finished product inspection

If you choose to test the finished product, the cost is:

$$Cost(S_3 = 1) = det_{prod} \times num_{product} + Cost(S_4) \quad (10)$$

Among them det_{prod} is the cost of testing the finished product.

If the finished product is not tested, the cost is:

$$Cost(S_3 = 0) = p_{prod} \times L_{return} \times num_{product} + Cost(S_4) \quad (11)$$

where p_{prod} is the defective rate of the finished product.

State transfer equations:

$$Cost(S_3) = \min(Cost(S_3 = 1), Cost(S_3 = 0)) \quad (12)$$

Stage 2: Part 2 Inspection

If Inspection Part 2 is selected:

$$Cost(S_2 = 1) = det_2 \times num_{product} + Cost(S_3) \quad (13)$$

where det_2 is the cost of testing part 2.

If part 2 is not tested:

$$Cost(S_2 = 0) = p_2 \times L_{return} \times num_{product} + Cost(S_3) \quad (14)$$

where p_2 is the defective rate of part 2.

State transfer equations:

$$Cost(S_2) = \min(Cost(S_2 = 1), Cost(S_2 = 0)) \quad (15)$$

Stage 1: Part 1 Inspection

If you choose to detect part 1:

$$Cost(S_1 = 1) = det_1 \times num_{product} + Cost(S_2) \quad (16)$$

where det_1 is the cost of testing part 1.

If part 1 is not tested:

$$Cost(S_1 = 0) = p_1 \times L_{return} \times num_{product} + Cost(S_2) \quad (17)$$

where p_1 is the defective rate of part 1, and L_{return} is the replacement loss.

State transfer equations:

$$Cost(S_1) = \min(Cost(S_1 = 1), Cost(S_1 = 0)) \quad (18)$$

Through this modeling process, we simulate the enterprise's decision-making, listing the costs at each production stage using reverse recursion. The four-step cascade covers the entire process from spare parts to finished products and after-sales, balancing cost and benefit in a dynamic environment. Verifying the cost-benefit relationship in different contexts demonstrates the hypothesis's adaptability and optimization value.

4 Results

According to the model established above, this paper obtains the minimum total cost, i.e., the optimal decision-making scheme, by substituting the specific parameter values of the six scenarios of the enterprise's production, and writing a program in python to perform the calculations, and continuously recursively.

The final results are shown in Table 1:

Table 1 Decision-making scenarios by scenario

	Situation 1	Situation 2	Situation 3	Situation 4	Situation 5	Situation 6
Part 1 Inspection	F	F	T	T	F	F
Part 2 Inspection	F	F	F	T	T	F
Finished Product Inspection	F	F	F	T	F	F
Defective Products' Disposal	T	T	T	T	T	F

From the table, we can analyze the cost, revenue, net profit and decision paths in detail for different scenarios, and the specific calculated values for different scenarios are shown in Table 2:

Table 2 Calculation results

Situation	Maximum Net Profit (Yuan)	Total Cost (Yuan)	Total Revenue (Yuan)	Part 1 Inspection	Part 2 Inspection	Finished Product Inspection	Defective Products' Disposal
Situation 1	20.10	30.30	50.40	F	F	F	T
Situation 2	12.20	32.60	44.80	F	F	F	T
Situation 3	13.90	36.50	50.40	T	F	F	T
Situation 4	11.80	33.00	44.80	T	T	T	T
Situation 5	18.90	31.50	50.40	F	T	F	T
Situation 6	23.70	29.50	53.20	F	F	F	F

As shown in Table 2, case 6 achieves the highest net profit. This indicates that avoiding multiple tests and treatments in production leads to a small negative impact from defective products, as the associated costs are low. Therefore, firms can increase net profit by adopting cost-reducing strategies.

As inspections increase, net profit gradually declines. In case 4, comprehensive testing and recycling reduce the defect rate but significantly raise costs, leading to the lowest net profit. Overall, when inspection and disposal costs are high and the defect rate is low, firms achieve higher net profitability by avoiding additional inspections.

5 Hypothesis testing

5.1 Hypothesis Testing Establishment

To ensure the model's applicability to real scenarios, this paper simulates production environments using hypothesis testing, applies Bayesian correction, and re-establishes the decision-making model. Comparisons are then made to test the validity of the dynamic planning model.

For spare parts, since the defective rate of spare parts is given as the defective x of the sample spare parts obtained from the sampling test, we need to get the range of the defective rate of the actual spare parts p . Using a hypothesis test with a confidence level of 95% and a Z-test, solve for the maximum acceptable value of p as p_0 , which will be the actual defective rate of the defective parts.

Null hypothesis:

$$H_0: p > p_0 \quad (19)$$

Alternative hypothesis:

$$H_1: p > p_0 \quad (20)$$

Select the level of significance, $\alpha = 1 - 0.95 = 0.05$.

The rejection domain was then determined. According to the significance level α and the type of test, the rejection domain formula is

$$\frac{\bar{x} - p_0}{\frac{\sigma}{\sqrt{n}}} \geq Z_\alpha \quad (21)$$

Where, Z_α is the critical value of the standard normal distribution at the significance level $\alpha = 0.05$, the specific value of Z_α is obtained by looking up the table for n , which is the size of the sample taken; we specify $\sigma = 0.02$.

The solution p_0 is 0.1010, that is to say, in 95% confidence, if the defective rate of spare parts is more than 0.1010, the batch of spare parts will be rejected, so we will consider the actual defective rate of spare parts.

Assuming a sample size of $n=2000$, we can use the same method to calculate the actual defective rate.

5.2 Bayesian formulation modeling

$$P(A|B) = P(B)P(B|A) \cdot P(A) \quad (22)$$

(1) Spare parts

It is known that the sample size $n = 2000$, standard deviation $\sigma = 0.02$, nominal defective rate $p_0 = 0.100735$, significance level $\alpha = 0.05$ and corresponding critical value $Z_\alpha = 1.645$.

Standard error calculations were first performed and the standard error SE

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{0.02}{\sqrt{2000}} = \frac{0.02}{44.72} \approx 0.000447 \quad (23)$$

Subsequently using the z-test formula

$$Z = \frac{\bar{x} - p_0}{SE} \quad (24)$$

Rearranging the formula to solve for the actual defect rate $\bar{x}$

$$\bar{x} = Z \times SE + p_0 \quad (25)$$

Plugging in the Z-value and standard error, the actual defective rate was calculated using a significance level of 95% corresponding to $Z_{\alpha} = 1.645$ $\bar{x} = 1.645 \times 0.000447 + 0.10 = 0.100735$

(2) Semi-finished products1,2

The defective rate of parts $\bar{x}$ is 10.0735%, each semi-finished product1,2 consists of three parts and the three parts are independent of whether they pass or fail.

According to the Bayesian formula, the following probabilities need to be calculated to compute the rate of semi-finished 1,2 defective products:

The first is the probability that all three parts pass.

For semi-finished 1:

$$P\{\text{All three parts passed}\} = P\{\text{Part 1 qualified}\} \times P\{\text{Part 2 qualified}\} \times P\{\text{Part 3 qualified}\} \quad (26)$$

For semi-finished 2:

$$P\{\text{All three parts passed}\} = P\{\text{Part 4 qualified}\} \times P\{\text{Part 5 qualified}\} \times P\{\text{Part 6 qualified}\} \quad (27)$$

Because the same pass rate for each part is equal to $P\{\text{Parts qualified}\} = 0.8992645$

So $P\{\text{All three parts passed}\} = 0.727849$

Next, it is necessary to find the probability that the semi-finished product passes given that all three parts pass.

From the previous discussion, given that all three parts pass, assume that the semi-finished product has a pass probability of $P\{\text{Semi-finished products1,2 qualified}|\text{All three parts passed}\}$, usually we set this value to 1. This is because if all the parts pass, the semi-finished product will also be considered as pass-ing, i.e:

$$P\{\text{Semi-finished products1,2 qualified}|\text{All three parts passed}\} = 1. \quad (28)$$

The final probability that all three parts are needed to pass, i.e.

$$P\{\text{All three parts passed}|\text{Semi-finished products1,2 qualified}\} \quad (29)$$

Semi-finished products 1,2 pass can generally be considered as 1, because if semi-finished products 1,2 pass, then three parts must pass.

According to the Bayesian formula, the

$P\{\text{Semi-finished products1,2 qualified}\} =$

$$\frac{P\{\text{Semi-finished products1,2 qualified}|\text{All three parts passed}\} \times P\{\text{All three parts passed}\}}{P\{\text{All three parts passed}|\text{Semi-finished products1,2 qualified}\}} \quad (30)$$

$$= 1 \times 0.727849 = 0.727849$$

The defective rate of semi-finished products1,2 $S_{\text{Semi-finished products1,2}}$ is:

$$S_{\text{Semi-finished products1,2}} = 1 - P\{\text{Semi-finished products1,2 qualified}\} = 1 - 0.727849 = 0.272151 \quad (31)$$

After recalculation using Bayesian formula, the defective rate of semi-finished products 1,2 is 27.22%.

(3) Semi-finished products3

The defective rate of the parts is 10.0735%, semi-finished product 3 consists of two parts, and the pass rate of semi-finished product 3 depends on the probability that both parts pass.

Step1: Calculate the probability that both parts pass the test

The pass rate of semi-finished product 3 is the probability that both parts pass.

According to the principle of Bayes' formula, if the conformity of two parts is an independent event, the conformity rate of semi-finished product 3 can be expressed as:

$$P\{\text{Both parts passed}\} = P\{\text{Part 1 qualified}\} \times P\{\text{Part 2 qualified}\} \quad (32)$$

Since the pass rate for both parts is the same as: $P\{\text{Parts qualified}\} = 0.8992645$, the

the reason why $P\{\text{Both parts passed}\} = 0.8992645 \times 0.8992645 = 0.808676$

Step2: Calculate the pass rate of semi-finished product 3

We assume that semi-finished product 3 must pass when both parts pass, the

Therefore. $P\{\text{Semi-finished products 3 qualified}|\text{Both parts passed}\} = 1$

Semi-finished product 3 passes $P\{\text{Semi-finished product 3 passes}\} =$

$$P\{\text{Semi-finished products 3 qualified}\} = P\{\text{Both parts passed}\} = 0.808676 \quad (33)$$

Step3: Calculate the defective rate of semi-finished product 3 $S_{\text{Semi-finished products 3}}$ as

$$S_{\text{Semi-finished products 3}} = 1 - P\{\text{Semi-finished products 3 qualified}\} = 1 - 0.808676 = 0.191324 \quad (34)$$

The defective rate of semi-finished product 3 is 19.13%.

(4) Finished products

Similarly according to the Bayesian formula, the pass rate of the finished product can be expressed as:

$$P\{\text{Finished product qualified}\} = \frac{P\{\text{Finished product qualified}|\text{All three semi-finished products passed}\} \times P\{\text{All three semi-finished products passed}\}}{P\{\text{All three semi-finished products passed}|\text{Finished product qualified}\}} \quad (35)$$

$$= 1 \times 0.385526$$

The defective rate of finished products $S_{\text{Finished product}}$ is

$$S_{\text{Finished product}} = 1 - P\{\text{Finished product qualified}\} = 1 - 0.385526 = 0.614474 \quad (36)$$

5.3 Hypothesis testing and Bayesian modified model solution

Table 3 Calculated results of hypothesis testing

state of affairs	Maximum Net Profit (\$)	Total cost (\$)	Gross proceeds (\$)	Component 1 testing	Component 2 testing	Finished Product Inspection	Dismantling of sub-standard products
Situation 1	20.04	30.31	50.35	clogged	clogged	clogged	be
Situation 2	12.20	32.60	44.80	clogged	clogged	clogged	be
Situation 3	13.85	36.50	50.35	be	clogged	clogged	be
Situation 4	17.85	32.50	50.35	be	be	be	be
Situation 5	18.84	31.51	50.35	clogged	be	clogged	be
Situation 6	19.33	31.02	50.35	clogged	clogged	clogged	clogged

Maximized profit occurs in case 1 with a net profit of 20.04.

The re-estimated profit maximization scenarios are similar to the original ones, indicating the dynamic programming model's practical applicability. For example, cases 1 and 6 remain optimal, with minor adjustments in net profit (e.g., case 1 from 20.10 to 20.04). This suggests that the test rate adjustment had a slight impact but didn't change the overall decision path. Thus, dynamic planning models are crucial for production planning in enterprises.

6 Summary

In this paper, we study the decision-making problem under various production scenarios in the production process of an enterprise, focusing on the inspection of spare parts and finished products and the treatment of nonconforming products to make an optimization analysis. The decision paths are optimized to minimize the production cost by using a dynamic programming model in the reverse direction of the production process. The results show that different inspection and dismantling strategies will affect the total cost and profit of the enterprise, and each interconnected production link will affect the economic efficiency of the whole enterprise production, which needs to be considered by the enterprise in the whole production link.

Meanwhile, the findings of this paper verify the reliability of the dynamic planning model in complex production decision-making. By reasonably adjusting the detection and processing strategies, enterprises can improve the flexibility and accuracy of decision-making in the dynamically changing market environment. Future work can further extend the model and apply it to more complex production scenarios to help enterprises optimize production decisions in the face of uncertain market demand and supply chain fluctuations. And with the global emphasis on sustainable production and environmental protection, future research can consider more social and environmental factors in the model. By balancing economic efficiency and environmental protection in decision making, companies can not only reduce production costs, but also comply with the requirements of social responsibility and green production, which can further improve their social image and long-term competitiveness.

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